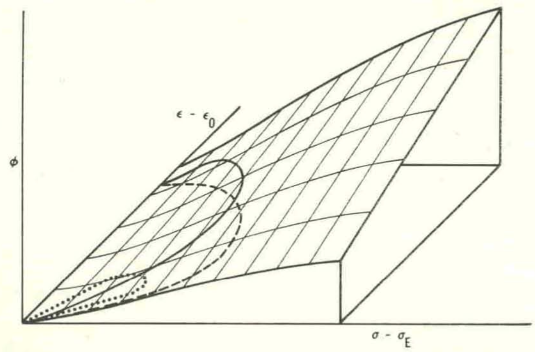

FIG. 10. Typical compaction surface $\dot{\epsilon} = \phi(\epsilon, \sigma - \sigma_E)$. The path on the surface shown as a heavy solid line is the locus of $(\epsilon, \sigma - \sigma_E, \dot{\epsilon})$ states taken at a material point during the passage of a wave. The dashed line is the projection of the path into the $(\epsilon, \sigma - \sigma_E)$ plane, and the small dotted loop is its projection into the $(\sigma - \sigma_E, \dot{\epsilon})$ plane.

recorded waveform, is the equilibrium stress-strain curve.

B. Collapse Rule

Inference of the function ϕ of Eq. (13) from a sequence of steady waveforms of various amplitudes and speeds is, in principle, a straightforward matter. Let us suppose that the data consist of several particle-velocity records, along with a measurement of V for each wave. The equilibrium curve is found as described above and can now be considered known. Using Eqs. (9), the known values of ρ_0 and V, and the known initial conditions, the stress and strain histories can be determined. Let us consider a given time t^*. From the stress history we read off $\sigma(t^*)$, and from the strain history $\epsilon(t^*)$ and $\dot{\epsilon}(t^*)$ can be determined. Since the function $\sigma_E(\epsilon)$ is known, we have values of $\dot{\epsilon}$, σ, and $\sigma - \sigma_E$ at $t = t^*$ and are thus able to plot a point of the surface ϕ as shown on Fig. 10. This process is repeated for a number of values t^* for each of the records in hand to map out a portion of the surface. From each experimental record we obtain values of ϕ associated with $(\epsilon, \sigma - \sigma_E)$ points lying on a curve in this plane (such as that shown by the dashed line in Fig. 10) that passes through the values ϵ_0 and ϵ_1 on the ϵ axis, and is single valued in ϵ. As an example we note that, in the case of a material governed by the linear collapse rule and the quadratic equilibrium behavior, these curves are parabolas with their maximum value, $(\sigma - \sigma_E)_{\max} = \rho_0 c_0^2 \beta^2 [\frac{1}{2}(\epsilon_1 - \epsilon_0)]^2$, taken at $\epsilon = \frac{1}{2}(\epsilon_1 + \epsilon_0)$.

Families of steady waveforms obtained by means of the usual plate-impact experiments have the same initial states (σ_0, ϵ_0), but different amplitudes. Since the Rayleigh lines corresponding to these waves do not cross, no two waves will have an $(\epsilon, \sigma - \sigma_E)$ point in common, so no conflicting values of ϕ can arise. By the same token, we see that the process described will always lead to a function ϕ that reproduces all of the observed waveforms exactly.

The possibilities for fitting less general collapse rules such as that of Eq. (12) or the specific forms in the examples of Sec. IV to experimental data are, of course, more limited. If the surface $\phi(\epsilon, \sigma - \sigma_E)$ determined by the means discussed above turns out to be a cylindrical sheet (in which case the generators will be parallel to the ϵ axis since this line is on the surface), then, of course, Eq. (12) is appropriate and the material is characterized by a curve in the $(\sigma - \sigma_E, \dot{\epsilon})$ plane.

As a practical matter, it may be desirable to restrict one's effort to fitting a simple collapse rule to available data. A collapse rule of the form of Eq. (12) is completely determined (over the range in question) by the highest-amplitude waveform in hand; it is just the locus of points $(\sigma - \sigma_E, \dot{\epsilon})$ obtained from this record. For most materials (idealized locking materials being the exception) $\sigma - \sigma_E$, and hence $\dot{\epsilon}$, vanish at both initial and final strains in the wave profile, and for this reason the locus of the $(\sigma - \sigma_E, \dot{\epsilon})$ points will form a closed path in this plane that begins and ends at the origin. An example of such a path is shown as the small dotted loop on Fig. 10. If the collapse rule of Eq. (12) is appropriate, the path will be a single line that is retraced for the upper portion of the wave profile. If, however, the path is a wide loop, a strong strain dependence is indicated and Eq. (12) does not provide an adequate model of the behavior. A less abstract check on the adequacy of the form is obtained by simply calculating lower-amplitude wave profiles and comparing them with experimental records. Collapse rules of such simple forms as those of Eqs. (14) and (26) can be established by choosing the coefficients T or T_1 and T_2, respectively, for best fit to the $(\sigma - \sigma_H, \dot{\epsilon})$ curve.

When the equilibrium response of the material is adequately represented by the locking model, steady-wave solutions are particularly simple and somewhat stronger statements can be made. For example, we see from Eq. (28) that a necessary condition for the applicability of the collapse rule of Eq. (22) is that the product $(\sigma_1 - \sigma_0)\mathscr{T}$ be the same for each member of a family of wave profiles. Let us suppose that this is true in some instance, and that we would like to fit the waveforms of Eq. (29) to the data. Since we have determined the constant value of the quantity $(\sigma_1 - \sigma_0)$, Eq. (30) becomes a relationship giving a one-parameter family of coefficient pairs $(T_0(\alpha^2), \alpha^2)$ for which all the waveforms given by Eq. (30) have the rise times observed in the experiments.

The remaining parameter α^2 can be adjusted to improve the agreement between the calculated and observed waveforms. As α^2 is increased the upper portion of the waveform is steepened with the lower portion being spread more to keep the rise time the same. When an approximate fit of a simple collapse rule to experimental observations is desired, the first thing to be decided upon is an appropriate criterion of goodness of fit. Usually one would like a reasonable fit to a range of waveforms rather than a perfect fit to some and a large error for others. For this purpose, one of the more reasonable criteria of good fit would be agreement between theory and experiment on the amplitude dependence of rise time.

VI. SUMMARY

In the previous sections of this paper, we have presented and discussed a simple theory of the dynamic compaction of porous solids. This theory elaborates the conventional theory of shock propagation in such a way that the observed shock structures can be described. It is not the only reasonable theory for this purpose, but does seem representative of several that could be proposed. We have shown that, for each material, the theory can be fit exactly to all steady-wave profiles having a given initial state. The same is true of several other theories in which the collapse rule involves two independent variables. Theories involving collapse rules that are special cases of Eq. (13) can, in general, be fit only approximately to experimental observations. A brief discussion of how this fitting could be accomplished has been presented. Examination of a variety of solutions such as were given in Sec. IV is helpful in deciding on the form of a collapse rule appropriate to fitting a specific set of data.

The conclusion that the collapse rule of Eq. (13) could be fit exactly to all steady-wave data following from the usual plate-impact experiment leaves open the question of how one can obtain a meaningful check of theory against experiment. From an examination of the discussion of Sec. V we see that what is needed is an experiment involving a compaction path that intersects the pencil of Rayleigh lines of the impact experiments. The two sorts of experiments that come to mind are those involving precompressed samples (so that the initial conditions are changed) and those in which evolving waves are studied. The former are reasonable and simple to perform. The latter also seem promising, but are more difficult since the theoretical predictions to be compared with the experiments must follow from solutions of the partial differential Eq. (5), along with appropriate constitutive equations.

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